UKFluids SRV: Fast solvers for structure-preserving discretisations of the stationary incompressible Navier–Stokes equations

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In this short visit, we added additional capabilities to the Firedrake finite element library[1] to support geometric multigrid schemes on non-nested hierarchies of meshes. This was a key step in extending the multigrid scheme of Benzi and Olshanskii [1] and Farrell, Mitchell, and Wechsung [2] to an exactly divergence-free element pair for the Navier–Stokes equations.

Within Firedrake itself, we added support in the multigrid code to construct the supermesh of intersecting cells (using the supermeshing library `libsupermesh`[2]). This enables the construction of restriction and prolongation operators on hierarchies of meshes that are geometrically related, but not necessarily related by regular (edge midpoint) refinement.

As well as improvements in Firedrake itself, we further extended the Schwarz preconditioning framework developed in part during a previous SRV. This new preconditioner for the PETSc library[3] has now been merged and is available in the most recent PETSc release.

The aim of the visit was to extend the preconditioning scheme of Farrell, Mitchell, and Wechsung [2] for the incompressible Navier–Stokes equations to a pressure-robust discretisation. That is, a discretisation for which errors in the pressure do not pollute the velocity solution. With the multigrid changes in place, we have been able to implement a discretisation of the Navier–Stokes equations using the Scott-Vogelius element pair that uses our multigrid scheme. The results are promising, with excellent convergence on problems up to a few million degrees of freedom in both two and three dimensions. We are now preparing scaling runs (with help from an ARCHER RAP allocation) to demonstrate the feasibility for high resolution simulations.

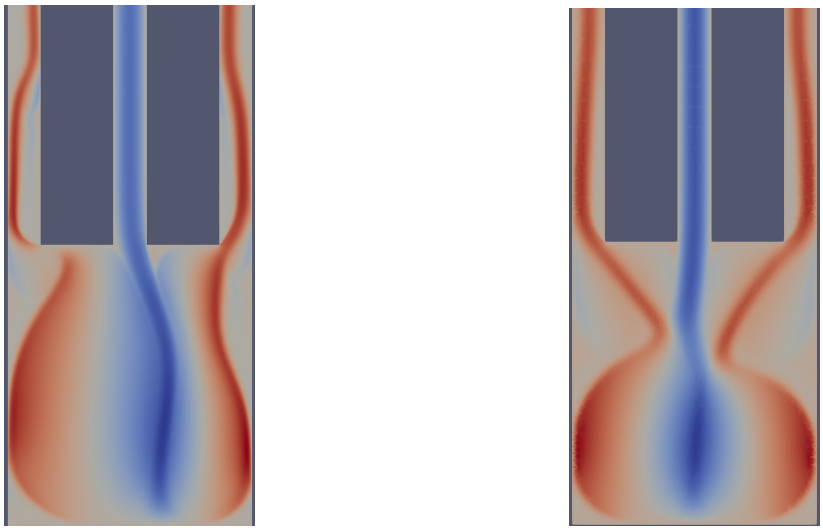

Figure 1: Two distinct solutions for laminar flow into a chamber. The new solver developed provides a significant speedup in the time to solution compared to a previous implementation in oomphlib. Images courtesy of Jessica Williams (Oxford)

[1] `www.firedrakeproject.org`

[2] `https://bitbucket.org/libsupermesh/libsupermesh`

[3] `www.mcs.anl.gov/petsc`

References

[1] M. Benzi and M. A. Olshanskii. "An Augmented Lagrangian-Based Approach to the Oseen Problem". In: *SIAM Journal on Scientific Computing* 28.6 (2006), pp. 2095–2113. DOI: 10.1137/050646421.

[2] P. E. Farrell, L. Mitchell, and F. Wechsung. *An augmented Lagrangian preconditioner for the 3D stationary incompressible Navier–Stokes equations at high Reynolds number*. Submitted to SIAM SISC. 2018. arXiv: 1810.03315 [math.NA].