Low-order modelling of thick films at moderate Reynolds number
UKFN SRV Report for Alexander Wray (AWW), University of Strathclyde
Visiting Radu Cimpeanu (RC) at the Oxford Centre for Industrial and Applied Mathematics

Aim of visit

The aim of the Short Research Visit was for the applicant to attend Oxford University for a week in order to collaborate on the problem of modelling thick liquid flows: in the past, low-order models for liquid layers have relied on the assumption that the layers are very thin (relative to the radius of curvature of the underlying substrate). This work built on existing research by AWW which alleviated this assumption, albeit only in the Stokes flow (zero Reynolds number) limit. The intention was to extend the work to incorporate inertial effects, and possibly other physical effects.

Results

The SRV was more successful than could have been hoped; in brief it is anticipated that in the immediate future it will result in at least two if not three papers. In addition it has formed part of the underpinning of one successful PhD proposal, and has resulted in a second which is currently under consideration (shortlisted) by the Carnegie Trust.

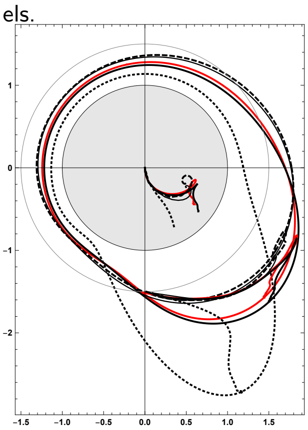
Inertial results

We examined the Moffatt problem: the 2-dimensional flow of a liquid layer hanging from a horizontal cylinder. This includes gravity, capillarity, viscosity and inertia, as well as a rotational base flow. Previously models had been confined to layers with thickness $\sim 10\%$ of the radius of the cylinder.

RC performed Direct Numerical Simulations while AWW developed and computed a variety of low-order models, as shown in Figure 1. In the figure, the new model is given by solid black lines while the DNS is given by red lines. The other lines constitute a variety of other models included for comparison (the new model incorporates two non-standard components, and the additional lines essentially demonstrate that both components are necessary, and that neither is sufficient in isolation).

The agreement is strong throughout parameter space, and the solution pleasingly tractable analytically. AWW and RC are currently writing this up with a view to submitting to Physical Review Fluids. As a result of this, AWW has recently taken on a PhD student to continue investigations in other geometries.

Figure 1: Liquid films hanging from a rotating cylinder: interface position, centre of mass and maximal thickness locations for exact solution (solid red line), new long-wave solution (solid black line) and a variety of simplified models.

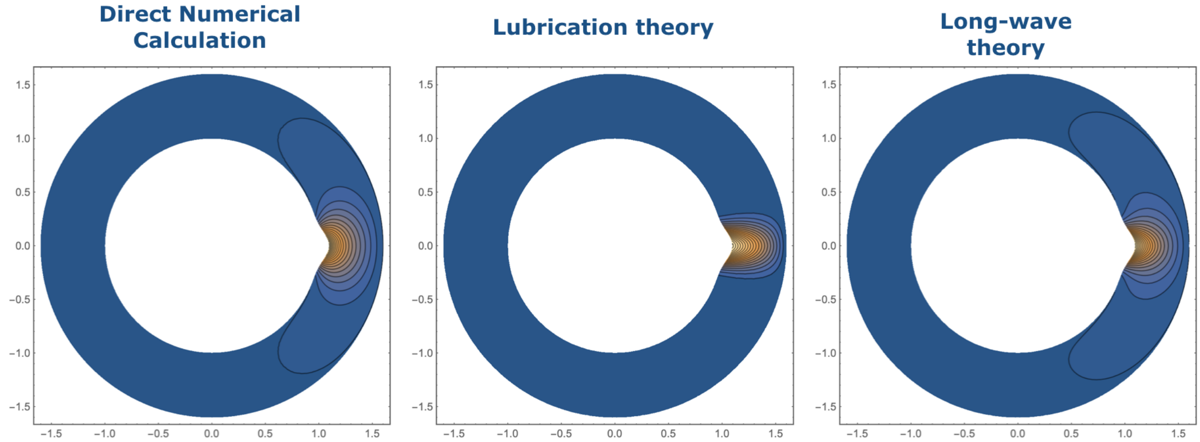


As part of the solution, a curious issue is raised about the leading order solution which turns out to be similar to, but fundamentally distinct from, the equivalent question in a gradient expansion approach. While of less direct fluid dynamical interest the problem is of great significance to modelling, and so RC and AWW have begun to prepare a second manuscript regarding this aspect of the model in particular.

Electrostatic results

An enormously helpful opportunity that is particular to an SRV is the additional time afforded in person, giving the chance to discuss broader ideas. One particular idea that we had the opportunity to discuss was other contexts where the solution technique might prove helpful. One particular issue

Figure 2: We compute Laplace's equation subject to $\phi = 0$ at the inner boundary, and $\phi = 1$ at the outer boundary. We plot contours of the difference between the computed solution, and the "basic solution" where the inner boundary is an exact circle. It is seen that the compact disturbance introduced to the inner boundary has a distinctly non-local impact that only the long-wave theory picks up.



with the classic "gradient expansion" commonly used in asymptotic models in fluid dynamics is that the solutions are fundamentally *local*: disturbances do not extend laterally outside the domain of the forcing effect, even if physically they should. A classic example of this is electric fields, where any perturbation to the field typically has a nonlocal effect.

Therefore, we recently applied the same long-wave solution technique to a perturbed electric field. The results are given in Figure 2. It is readily noted that the disturbances given by DNS and long-wave theory are fundamentally non-local, unlike that given by classic lubrication theory (gradient expansion approach). We plan to write this up (in particular exploiting it to control the Moffatt instability), and it has been used as part of an application submitted to the Carnegie Trust (currently shortlisted).