SRV report

Bath, 18 May 2018

Title: *"Development of multigrid preconditioners for hybridized Discontinuous Galerkin methods in atmospheric fluid dynamics"*

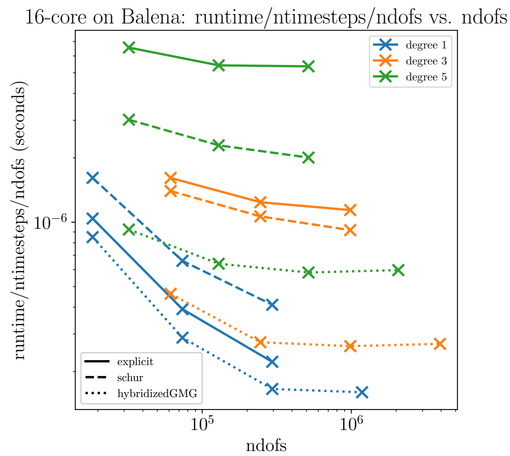
Outcomes of the visit

As planned, we used the time at Imperial College for discussions both on the numerical methods and their implementation in the code. Here in particular Dr. Lawrence Mitchell's in-depth knowledge of the Firedrake library was extremely helpful. Prior to the visit we had already implemented a simple semi-implicit timestepping scheme for the shallow water equations with the Hybridized Discontinuous Galerkin (HDG) method. This had been the result of an existing collaboration with Thomas Gibson (Imperial College). However, this code had been unrealistically simple.
Based on this previous work, the following was achieved during the visit:

- **Improved implementation of a geometric multigrid preconditioner**
  A key ingredient of the HDG method is an efficient solver for the DG flux system. Although our code already contained a first version of the non-nested Geometric Multigrid algorithm, similar to the one in [1], the implementation was very slow. During the visit Lawrence Mitchell re-implemented the algorithm and made it more configurable, which will be important for proper testing to systematically identify the best solver settings and ultimately produce a competitive semi-implicit timestepping method.
- **Clarification of the timestepping scheme for the full, non-linear equations**
  So far we had only implemented the linearised shallow water equations with simplified time integrators (an implicit θ-method combined with explicit integration of the Coriolis force). To make realistic predictions, advanced time stepping methods such as the IMEX algorithms in [2] have to be used for the full, non-linear equations. Through discussions with our colleagues we were able to clarify the structure of the discrete equations for the full model and the efficient implementation of the methods in our Firedrake code. Our discussions will help to avoid inefficiencies which would arise in a naive implementation of advanced timestepping methods.
- **Improved implementation of shallow water equations in a spherical geometry**
  By using a mapping from suitable two-dimensional finite elements it was possible to construct a purely tangential velocity field for spherical geometries. This reduces the number of unknowns since only the tangential components is stored; we implemented this improved scheme in our code.

Next steps

Currently it looks like we can achieve significant speedups relative to a native DG discretisation (see plot below), thanks in part to the work during the SRV. Continuing on from the discussions during the visit currently implement realistic timestepping methods for the full non-linear equation set and intend to publish the results of runs for larger problems on an HPC installation in a paper within the next months.



Successful discussions on the sun-terrace of Imperial's physics building (left) and measured runtimes achieved with the new hybridised GMG method (right). For all polynomial degrees the new method (dotted lines) is significantly faster than the previous Schur-complement preconditioner (dashed lines) and an explicit timestepping method (solid lines).

References

[1] Gopalakrishnan, J. and Tan, S., 2009. Num. Lin. Alg. with Appl., 16(9), pp.689-714.
[2] Kang, S., Giraldo, F.X. and Bui-Thanh, T., 2017. arXiv:1711.02751.