Nonlinear Waves and Interfacial Dynamics, 22-23 June 2018, Loughborough, UK
Programme and abstracts

Day 1 - Friday, 22 June 2018 (Lecture Room SCH001)

12.00 -- 13.00 **Registration and lunch (SCH004)**

Chair: TBC
13.00 – 13.40 *Demetrios Papageorgiou (Imperial College)*
Nonlinear instabilities in multilayer flows

13.40 – 14.20 *Jonathan Healey (Keele University)*
Fractal sets of neutral curves in stably stratified plane Couette flow

14.20 – 15.00 *Sara Lombardo (Loughborough University)*
Nonlinear wave instabilities via integrability

15.00 – 15.30 ***Coffee/Tea break***

Chair: TBC
15.30 – 16.10 *John Billingham (University of Nottingham)*
A dam-break in a crossflow

16.10 – 16.50 *Gennady El (Loughborough University)*
Nonlinear Schroedinger equation and the universal description of dispersive shock waves

16.50 – 17.30 *Sergey Annenkov (Keele University)*
Long-term spectral evolution of water waves: direct numerical simulations vs kinetic equations modelling and observations

17.30 – 18.00 ***Coffee/Tea break***

Chair: TBC
18.00 – 19.00 ***Open Lecture***
Wooyoung Choi (New Jersey Institute of Technology, USA)
Nonlinear waves and their interactions in the ocean

19.00 – ***Reception or Trip to Carvery (to be confirmed)***

Day 2 - Saturday, 23 June 2018 (Lecture Room J001)

Chair: TBC
10.00 – 10.40 *Magda Carr (Newcastle University)*
Fully nonlinear internal solitary waves

10.40 – 11.20 *Marco Discacciati (Loughborough University)*
An introduction to domain decomposition methods

11.20 – 12.00 *Dmitri Tseluiko (Loughborough University)*
Discrete self-similarity and formation of iterated patterns in interfacial hydrodynamics

12.00 – 13.00 ***Lunch and ECR and PhD students Poster Display***

ECR and PhD students Training Session

13.00 – 14.00 *Karima Khusnutdinova (**Introduction to KdV and Ostrovsky equations – theory) (J001)***

14.00 – 15.00 *Matthew Tranter (**Introduction to KdV and Ostrovsky equations – pseudospectral techniques in MATLAB) (IT Lab YY009)***

15.00 – Poster Prize and Close

Abstracts

Demetrios Papageorgiou (Imperial College)

Nonlinear instabilities in multilayer flows

This talk will be in two parts. The first will describe some recent results on the stability of multilayer shear flows of three or more immiscible viscous fluids caused to flow by gravity and/or a pressure gradient. There are now at least two free boundaries and asymptotic solutions will be described that yield a system of coupled partial differential equations for the interfacial positions. Some subtleties in the weakly nonlinear asymptotics will be pointed out. The equations generically support instabilities even at zero Reynolds numbers and these emerge physically from a resonance between the interfaces and manifest themselves mathematically through hyperbolic to elliptic transitions of the flux part of the equations. We use the theory of 2x2 systems of conservation laws to derive a nonlinear stability criterion that can tell us whether a system which is linearly stable, can (i) become nonlinearly unstable, i.e. a large enough initial condition produces a large time nonlinear response, or (ii) remains nonlinearly stable, i.e. the solution decays to zero irrespective of the initial amplitude of the perturbation. Having described weakly nonlinear solutions we turn to fully nonlinear deformations are also in the large surface tension limit giving rise to coupled Benney-type equations. Their fluxes also support hyperbolic-elliptic transitions and numerical solutions will be described giving rise to intricate nonlinear stable traveling waves. Differences between weakly and strongly nonlinear flows will be described to guide ideas for theory of coupled Benney equations.

The second part is concerned with three-dimensional instabilities of electrified falling film flows and in particular the multidimensional electrified Kuramoto-Sivashinsky equation which is the first in a hierarchy of nonlinear models. We will mostly present computations and end with words of caution in using 2D models instead of 3D ones in the presence of electric fields.

Jonathan Healey (Keele University)

Fractal sets of neutral curves in stably stratified plane Couette flow

Plane Couette flow is famous for being stable to small amplitude disturbances. We consider the linear stability of plane Couette flow between horizontal plates where the fluid density increases monotonically with depth, i.e. stably stratified flow. Perhaps surprisingly, the presence of stabilizing buoyancy forces can destabilize the flow. This instability mechanism can be understood by studying the Taylor-Goldstein equation, a linear second order ordinary differential equation. We consider some simple model density profiles. In fact, they are sufficiently simple that explicit solutions in terms of special functions can be written down. However, the properties of the roots of the dispersion relation are nontrivial.

The strength of the density gradient at a point in the flow compared to the strength of the shear is characterized by a dimensionless parameter, the local Richardson number, Ri. A theorem due to Howard and Miles proves that a necessary condition for instability is that Ri < 1/4 somewhere in the flow. We introduce a parameter plane for our density profile where one axis gives the location in the flow of the point where Ri takes its minimum value, and the other axis characterizes the total change in density across the flow (a bulk, or global, Richardson number).

We show that when the minimum value of Ri in the flow domain is 1/4, an infinite number of points appear in the parameter plane at which neutral waves exist. These points lie on a fractal set. As soon as the minimum Ri drops below 1/4, each neutral point opens up into a neutral curve, with each neutral curve forming a closed loop. The neutral loops expand and overlap one-another as Ri is reduced further, giving a fractal set of neutral curves. When the minimum Ri is reduced to zero, a fractal set of branch-points is encountered where neutral modes coalesce to form complex conjugate pairs.

Fractals are usually associated with nonlinear processes, but fractal behaviour has been found here in a linear stability equation that has been studied since the 1930s. Qualitatively similar behaviour can be expected for other density profiles.

Sara Lombardo (Loughborough University)

Nonlinear wave instabilities via integrability

The problem of stability is central to the entire field of nonlinear wave propagation and is a fairly broad subject. In this talk I am specifically concerned with the early stage of amplitude modulation instabilities due to quadratic and cubic nonlinearities and we consider in particular dispersive propagation in a one dimensional space, or diffraction in a two dimensional space. I will consider the linear stability properties of NLS-type systems and their continuous wave solutions. I will revisit the scalar NLS equation in this context and consider then the integrable coupling of two nonlinear Schr\"{o}dinger equations. Using the integrable properties of the system, one can compute and classify the so-called stability spectra, providing a necessary condition in the parameters space for the onset of instability. The derivation of the spectra is completely algorithmic and make use of elementary algebraic-geometry. It turns out indeed that, for a Lax Pair that is polynomial in the spectral parameter, the problem of classifying the spectra is transformed into a problem of classification of certain algebraic varieties. The method is general enough to be applicable to a large class of integrable systems. Notwithstanding this condition, it remains of important practical interest because several integrable partial differential equations have been derived in various physical contexts as reliable, though approximate, models. This work has been done in collaboration with Antonio Degasperis (Roma ``Sapienza'') and Matteo Sommacal (Northumbria).

John Billingham (University of Nottingham)

A dam-break in a crossflow

We study a flow that has been proposed as a very simple model for a powder snow avalanche (Carroll et al, Phys. Fluids, 066603, 2012). This is based on the inviscid, irrotational flow generated by a two-dimensional point source moving along a horizontal, impenetrable boundary. When the fluid that emanates from the source has the same density as the surrounding fluid, the flow is trivial. We consider what happens in an initial value problem that starts with the trivial flow, but with the source fluid of a different density to the surrounding fluid. We consider (i) the flow when the density difference is small, (ii) the small time asymptotic solution, which involves a dam-break at the head of the flow, (iii) numerical solutions.

Gennady El (Loughborough University)

Nonlinear Schroedinger equation and the universal description of dispersive shock waves

The nonlinear Schroedinger (NLS) equation and the Whitham modulation equations both describe slowly varying, locally periodic nonlinear wavetrains. The commonalities and differences between the two descriptions have been widely discussed in the literature. We use an overlap regime for the applicability of the NLS equation and the Whitham modulation theory to develop a universal analytical description of dispersive shock waves (also known as undular bores) for a broad class of integrable and non-integrable nonlinear dispersive equations.

Sergey Annenkov (Keele University)

Long-term spectral evolution of water waves: direct numerical simulations vs kinetic equations modelling and observations

Kinetic equations are widely used in many branches of science to describe the evolution of random waves spectra. To examine the validity of these equations, we study numerically the long-term evolution of water wave spectra using three different models. The first model is the classical kinetic (Hasselmann) equation (KE). The second model is the generalised kinetic equation (gKE), derived employing the same statistical closure as the KE but without the assumption of quasi-stationarity. The third model, which we refer to as the DNS-ZE, is a direct numerical simulation algorithm based on the Zakharov integrodifferential equation, which plays the role of the primitive equation for a weakly nonlinear wave field. It does not employ any statistical assumptions.

We perform a comparison of spectral evolution of the same initial distributions with and without wind forcing, with/without a statistical closure and with/without the quasi-stationarity assumption. For the initial conditions we choose spectra previously studied experimentally and numerically using a variety of approaches. Our DNS-ZE results are validated with the existing short-term DNS simulations by other methods and with the available laboratory observations of higher-order moment (kurtosis) evolution. Numerical simulations of long-term evolution of wind waves are compared with the available airborne measurements of wind wave field evolution.

We show that all models demonstrate very close evolution of integral characteristics of spectra. However, there are major differences between the DNS-ZE and gKE/KE predictions. First, the rate of angular broadening of initially narrow angular distributions is much larger for the gKE and KE, than for the DNS-ZE. Second, the shapes of frequency spectra differ substantially (even when the nonlinearity is decreased), the DNS-ZE spectra being wider than the KE/gKE ones and having a much lower spectral peak. Third, the maximal rates of change of the spectra obtained with the DNS-ZE scale as the fourth power of nonlinearity, which corresponds to the dynamical timescale of evolution, rather than the sixth power of nonlinearity typical of the kinetic timescale exhibited by the KE. The striking systematic discrepancies for a number of specific spectral characteristics call for the revision of the fundamentals of wave kinetic description. This is a joint work with Victor Shrira.

Wooyoung Choi (New Jersey Institute of Technology, USA)

Open lecture: Nonlinear waves and their interactions in the ocean

Nonlinear wave phenomena in the ocean are ubiquitous both on its surface and in its interior. Nevertheless, as the wave motions are often highly nonlinear and their spatial scales range from millimeters to thousands of kilometers, it is a nontrivial task to accurately model their generation, propagation, and dissipation. It is even more challenging when complicated interactions occur among waves of different spatial scales. In this talk, I will present recent efforts to better understand the underlying physics of highly nonlinear waves in the ocean through combined theoretical and experimental investigations and to develop reliable mathematical models to represent the evolution of such waves. Starting with surface waves possibly in the presence of external forcing and energy dissipation due to wave breaking, I will describe nonlinear internal waves in density-stratified oceans and their interactions with surface waves. Then, some remaining challenges and open questions will be discussed.

Magda Carr (Newcastle University)

Fully nonlinear internal solitary waves

Internal solitary waves (ISWs) are finite amplitude waves of permanent form that travel along density interfaces in stably stratified fluids. They owe their existence to an exact balance between non-linear wave steepening effects and linear wave dispersion. They are common in all stratified flows especially coastal seas, straits, fjords and the atmospheric boundary layer. In the ocean, they are thought to be a source of mixing, and are important in re-suspension of sedimentary materials, and mixing processes in the benthic boundary layer. They are subsequently of interest from both an environmental and offshore engineering point of view.

Numerical (contour advective semi-spectral) and experimental (sluice gate) techniques to model ISWs will be presented. A summary of past, present and future work by the speaker in this area will be given including (i) shear-induced instabilities in large amplitude ISWs, (ii) near bottom instabilities in the wave induced bottom boundary layer, (iii) mode-2 ISW characteristics and (iv) dynamics of ISW interaction with sea-ice.

Marco Discacciati (Loughborough University)

An introduction to domain decomposition methods

Domain decomposition methods are a class of mathematical and numerical techniques that allow to equivalently reformulate a given problem as a family of subproblems of reduced complexity suitably coupled one to another. The solution of the original problem can then be obtained by "gluing" the solutions of the subproblems. The development of these methods was strongly motivated by the spread of parallel computing in order to benefit from multiprocessor computers by assigning subproblems to different processors. However, recent research has shown that domain decomposition methods can also be successfully used to handle problems described by different equations in different regions of the domain of interest (multi-physics) and/or problems defined on geometrical domains of different dimensions (geometrical multi-scale). Thus, they provide a very powerful tool both for mathematical modelling and for numerical computations. In this talk, we will introduce the basic ideas of domain decomposition methods and we will show how they can be applied in various contexts.

Dmitri Tseluiko (Loughborough University)

Discrete self-similarity and formation of iterated patterns in interfacial hydrodynamics

Consider a thin liquid film coating a solid surface. Such a film may be destabilised by one of a number of physical effects. Examples include the Marangoni instability of a film heated from below, Rayleigh-Taylor instability of a film on a cylinder, and film instability due to intermolecular forces. Such instability may lead to a finite-time rupture of the film, which is described by a singularity of the solutions of the governing mathematical model. In the simplest scenario, such a singularity is of a self-similar nature. However, more complicated behaviour may arise, such as the formation iterated structures of geometrically shrinking droplets. We undertake a computational and theoretical study of the origin of such iterated structures using a lubrication-type model equation. We demonstrate that such structures appear as a consequence of discrete self-similarity, where certain patterns repeat themselves, subject to rescaling, periodically in a logarithmic time scale. We also reveal the mechanism for discrete self-similarity, and demonstrate that such behaviour appears as the result of a Hopf bifurcation from ordinarily self-similar solutions.