Numerical investigation on the dispersion of vaporizing polydisperse sprays in a gas flow

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Motivation

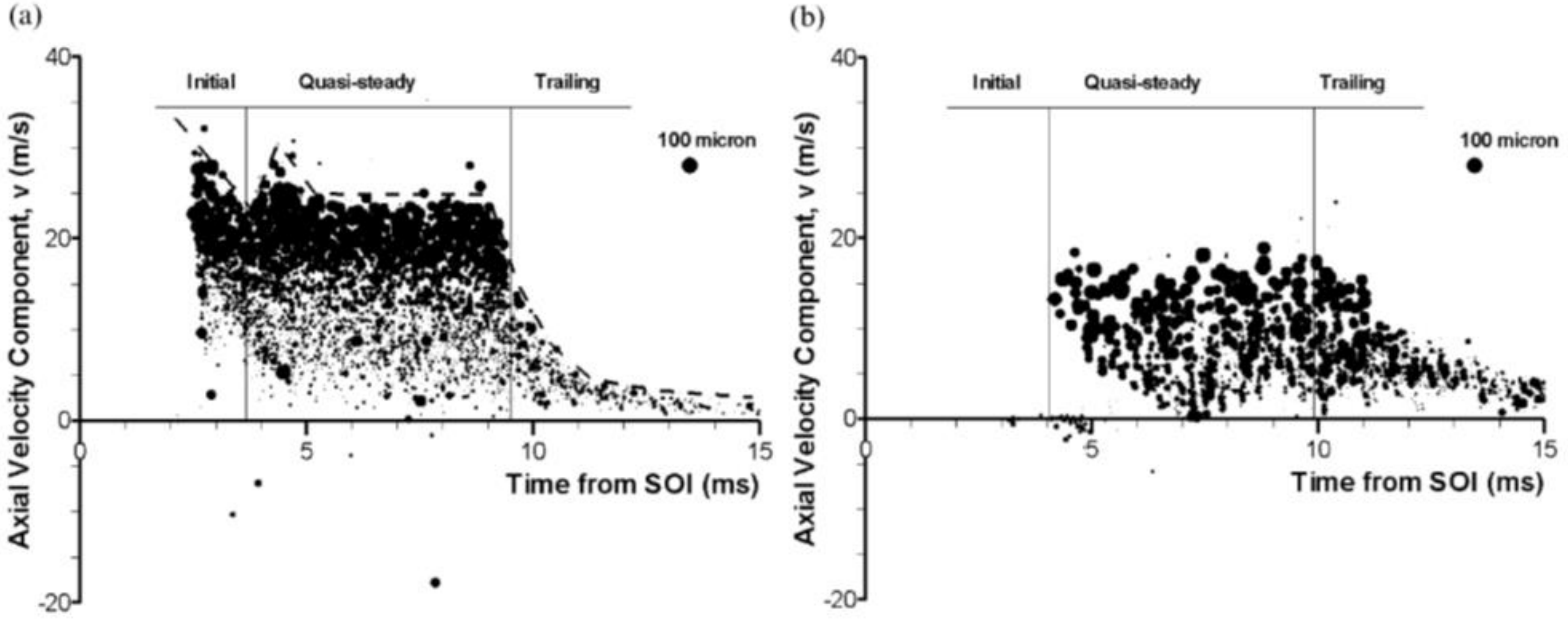

Fig. 3 Distribution of droplet diameters and velocities in the PFI injector spray plotted against time from SOI when (a) $r = 0$ mm and $x = 15$ mm and (b) when $r = 6$ mm and $x = 55$ mm

S. Begg, F. Kaplanski, S. Sazhin, M. Hindle, M. Heikal. *Int J. Engine Res*. 2009

- Polydisperse
- Evaporation

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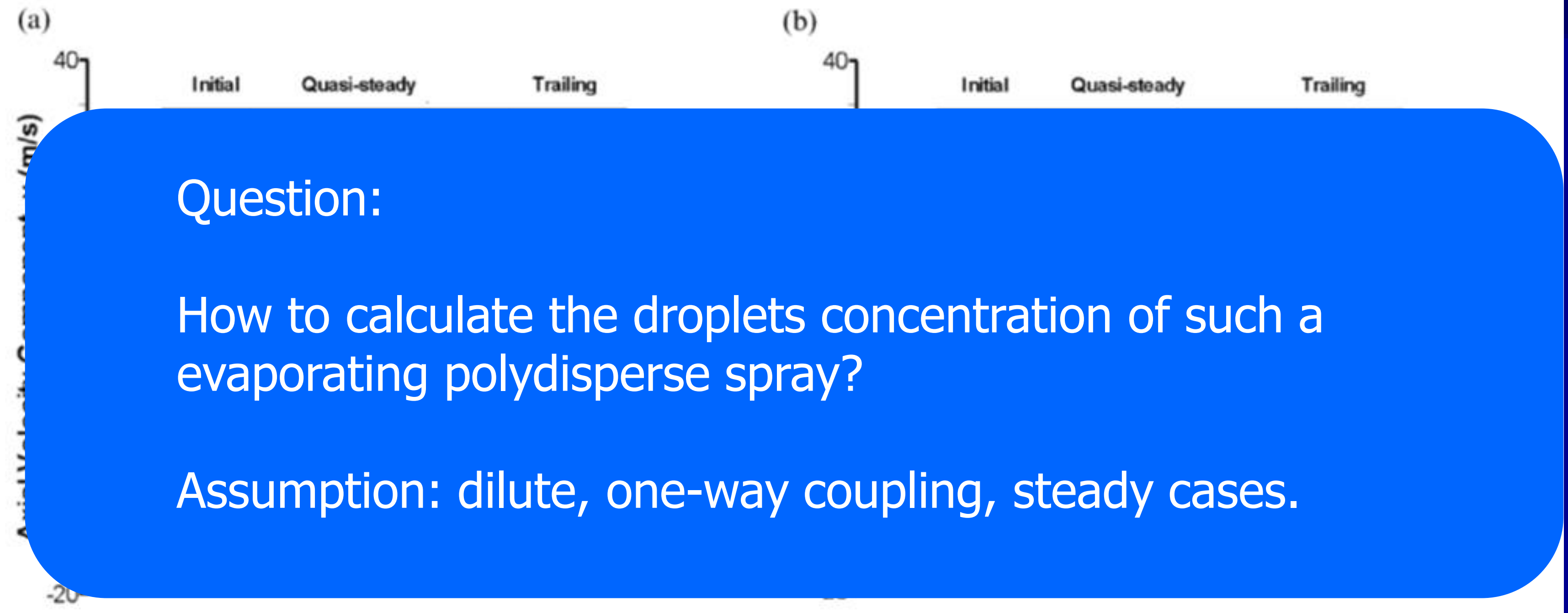

Question:

How to calculate the droplets concentration of such a evaporating polydisperse spray?

Assumption: dilute, one-way coupling, steady cases.

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- Polydisperse
- Evaporation

Mathematical framework: Fully-Lagrangian Approach(FLA)

Monodisperse spray with evaporation:

$$\rho_g \left(\frac{\partial}{\partial t} + \mathbf{u}_g \cdot \nabla \right) \mathbf{u}_g = -\nabla p + \mu_g \nabla^2 \mathbf{u}_g,$$
$$\nabla \cdot \mathbf{u}_g = 0,$$

One-way coupling carrier phase:
Navier-Stokes equations

$$\frac{d\mathbf{x}_d}{dt} = \mathbf{u}_d,$$
$$\frac{d\mathbf{u}_d}{dt} = \frac{\beta'}{r_d^2}(\mathbf{u}_g - \mathbf{u}_d), \quad \text{where} \quad \beta' = \frac{9\mu_g}{2\rho_d}$$
$$\frac{dr_d^2(t)}{dt} = -\alpha(T_g(t) - T_d), \quad \text{where} \quad \alpha = \frac{\lambda_d}{2\rho_d H}$$

Droplet phase:
- Lagrangian framework of the dynamics
- Droplet evaporation

$$n_d = \frac{n_0}{\|\mathbf{J}\|}, \quad \text{where} \quad J_{ij} = \frac{\partial \xi_i}{\partial \xi_{j0}} \quad \xi = (x_d, y_d, z_d, r_d)$$

Droplet phase:
Continuity equation

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$$n_d = \frac{n_0}{\|\mathbf{J}\|}, \quad \text{where} \quad J_{ij} = \frac{\partial \xi_i}{\partial \xi_{j0}}$$

$$\frac{dJ_{ij}(t)}{dt} = \omega_{ij}, \quad i = 1,2,3, \quad j = 1,2,3,4,$$
$$\frac{dJ_{4j}(t)}{dt} = \omega_{4j} = \frac{\alpha}{2r^2}(T_g - T_d)J_{4j} - \frac{\alpha}{r}\frac{\partial T_g}{\partial \xi_i} J_{ij},$$
$$\frac{d\omega_{ij}}{dt} = \frac{\beta'}{r_d^2}\left(\frac{\partial u_{g,i}}{\partial x_k} J_{kj} - \omega_{ij}\right) - \frac{2\beta'}{r_d^3}(u_{g,i} - u_{d,i})J_{4j} \quad i,k = 1,2,3, \quad j = 1,2,3,4.$$

Mathematical framework: Fully-Lagrangian Approach(FLA)

Polydisperse spray with evaporation:

Initial concentration varies for droplets at different initial size:

$$\tilde{n}_{d0} = \tilde{n}_{d0}(r_{d0})$$

↓ A monodisperse system with the given size and concentration

$$\tilde{n}_d = \tilde{n}_{d0} ||\tilde{\mathbf{J}}||^{-1}$$

↓

$$n_d = \int \tilde{n}_d dr_d$$

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$$\tilde{n}_{d0} = \frac{1}{r_{d0}} \frac{1}{S\sqrt{2\pi}} \exp\left(-\frac{(\ln r_{d0} - M)^2}{2S^2}\right)$$

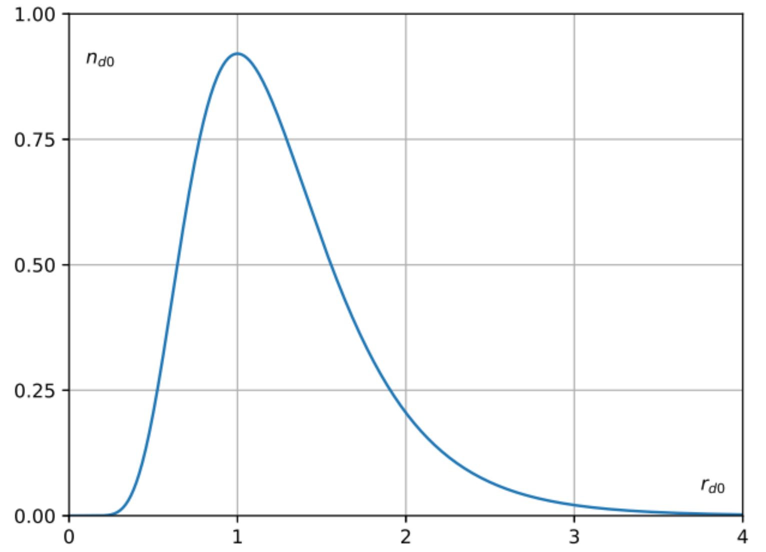

Figure 1: Initial distribution. M = 0.4, S = 0.16

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Question:

How to find which trajectory passes a chosen coordinate (x,y,z)?

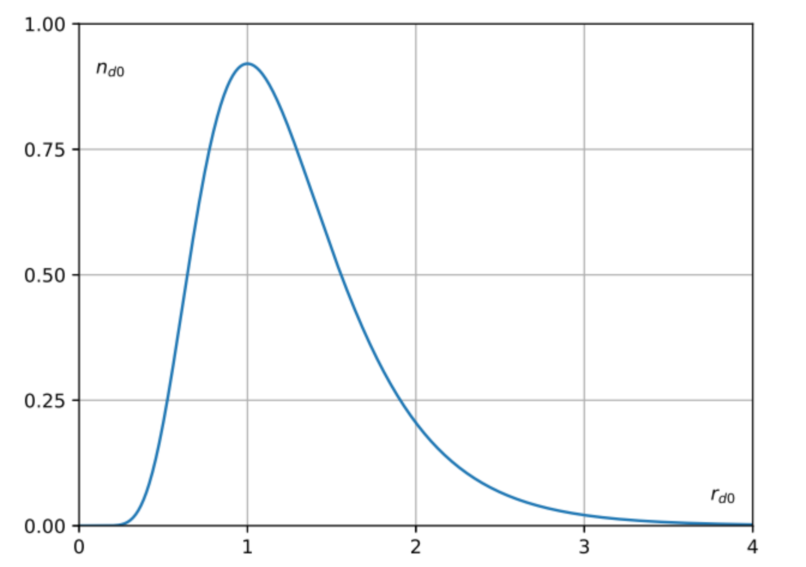

Figure 1: Initial distribution. M = 0.4, S = 0.16

Numerical task

FLA essentially have described:

$$(\mathbf{x}, \mathbf{u}, \tilde{r}_d, \tilde{n}_d) = (\mathbf{x}, \mathbf{u}, \tilde{r}_d, \tilde{n}_d)(t, \mathbf{x}_0, \mathbf{u}_0, \tilde{r}_{d0}, \tilde{n}_{d0})$$

Find $(t, \mathbf{x}_0, \mathbf{u}_0, \tilde{r}_{d0}, \tilde{n}_{d0})$ to satisfy $\mathbf{x} = \mathbf{x}_p$

Numerical treatment

Simplest case: Injection from one same point at the same velocity.

$$(\mathbf{x}, \mathbf{u}, \tilde{r}_d, \tilde{n}_d) = (\mathbf{x}, \mathbf{u}, \tilde{r}_d, \tilde{n}_d)(t, \tilde{r}_{d0})$$

Variables: $(\mathbf{x}, \mathbf{u}, \tilde{r}_d, \tilde{n}_d)$

Parameters: $(t, \tilde{r}_{d0})$

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Numerical scheme:

- Choose a number of samples at different initial sizes;

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- Choose a number of samples at different initial sizes;
- Using FLA framework to find the values of such variables along their own trajectories;

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Numerical scheme:

- Choose a number of samples at different initial sizes;
- Using FLA framework to find the values of such variables along their own trajectories;
- Define the interpolation rules, and find $(t, \tilde{r}_{d0})$ inversely satisfying $\mathbf{x} = \mathbf{x}_p$, thus interpolate the values of other variables at such parameters.

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Numerical scheme:

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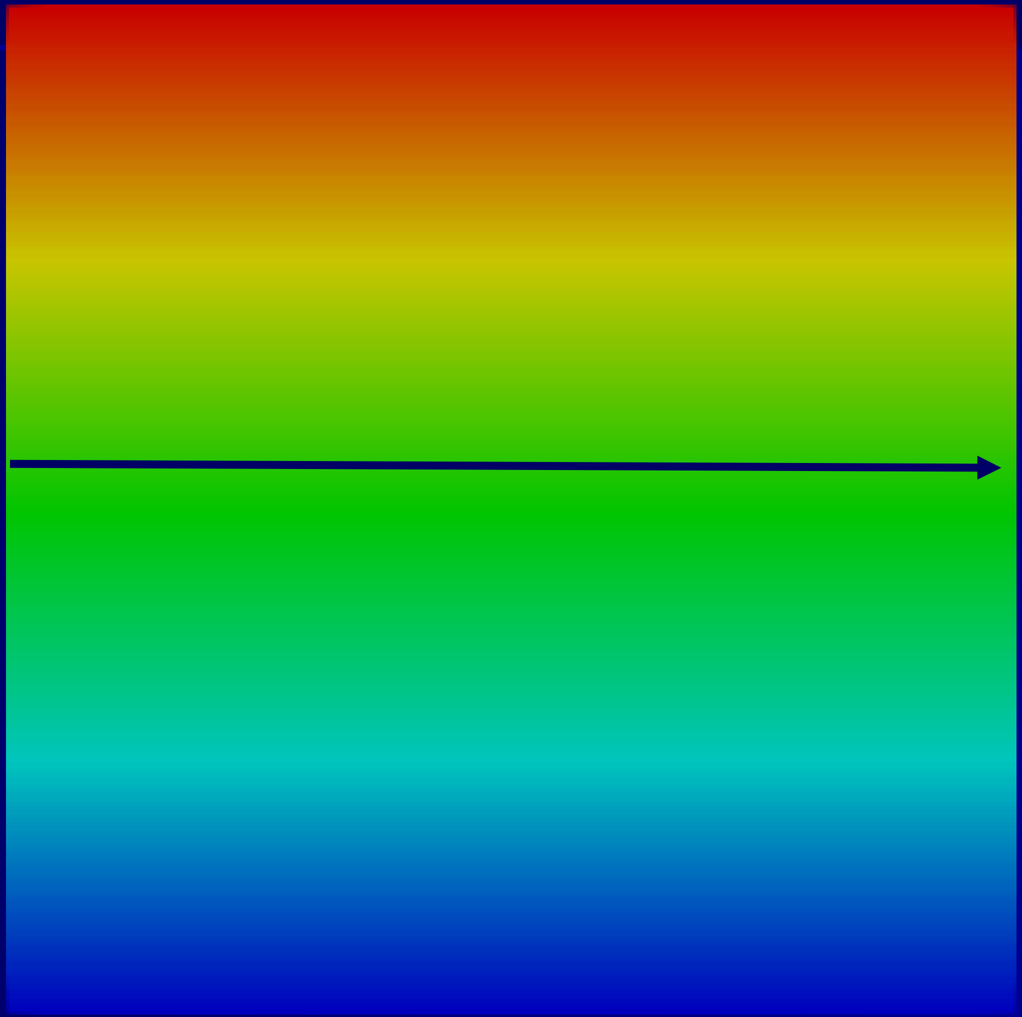
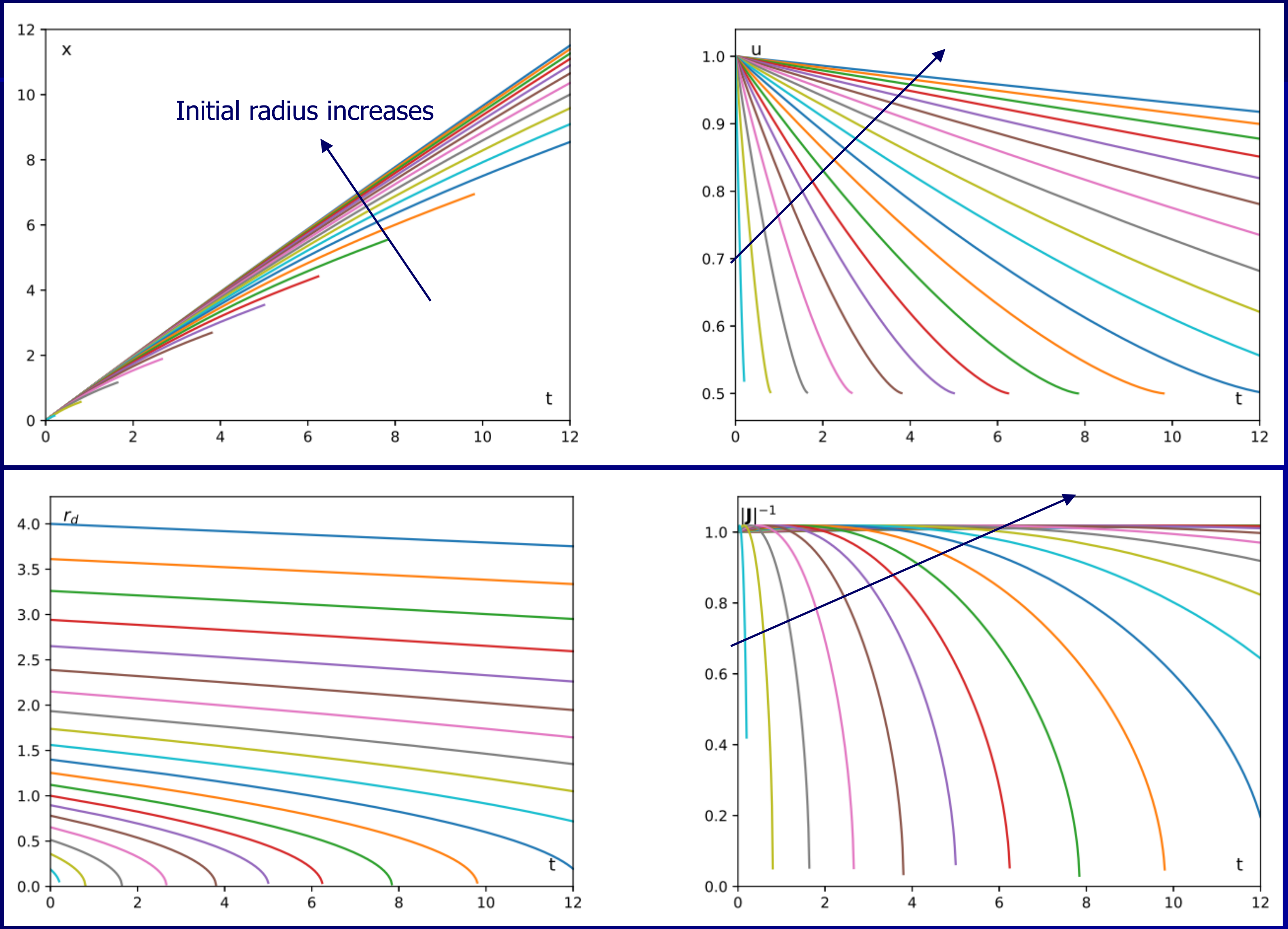
- Define the interpolation rules, and find $(t, \tilde{r}_{d0})$ inversely satisfying $\mathbf{x} = \mathbf{x}_p$, thus interpolate the values of other variables at such parameters.
- This method can be extended to the more complicated cases with more free parameters.

Examples I:
Horizontal injections into a hot planar-Couette flow



Horizontal injections: results



$$\tilde{n}_d = \tilde{n}_{d0} \|\tilde{\mathbf{J}}\|^{-1}$$

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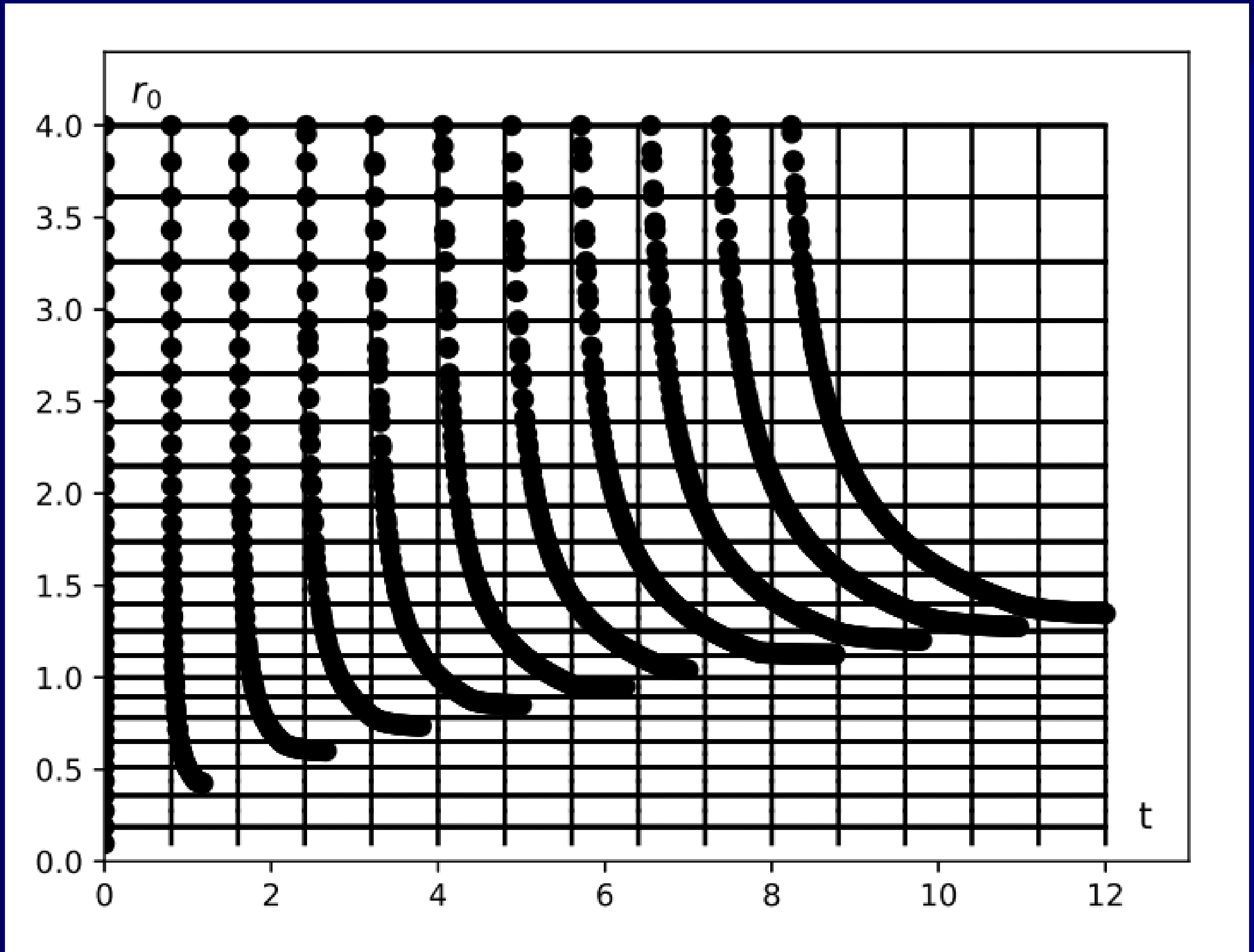
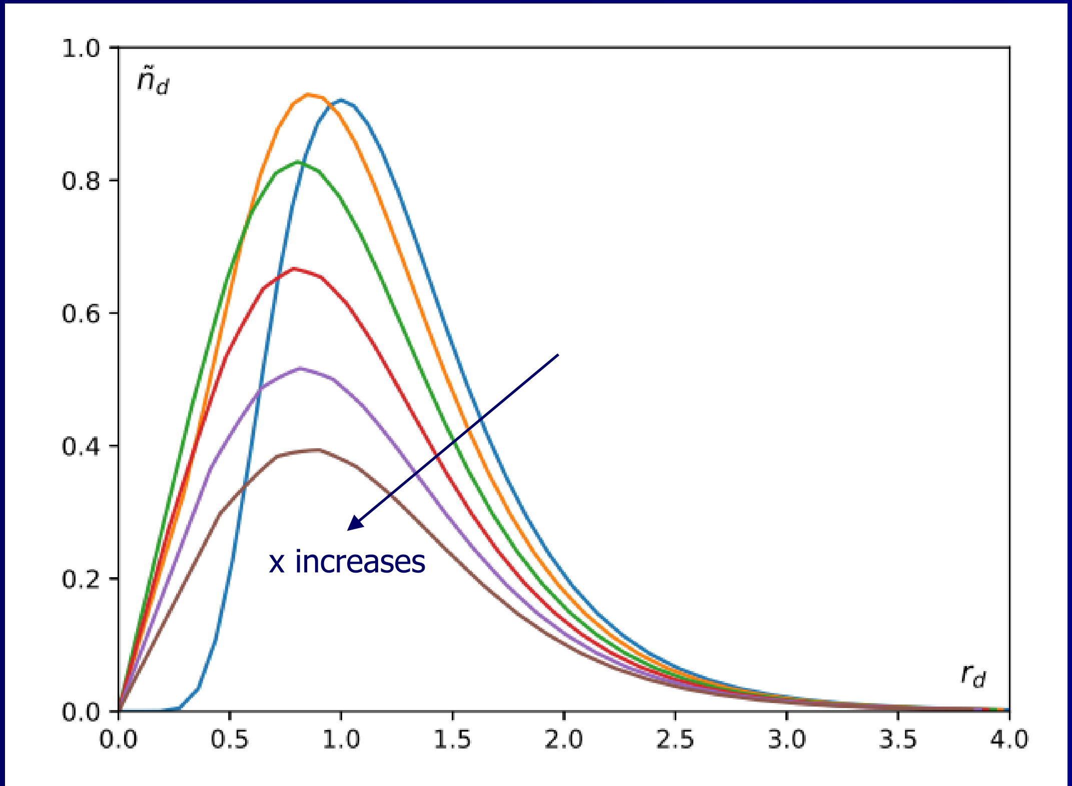
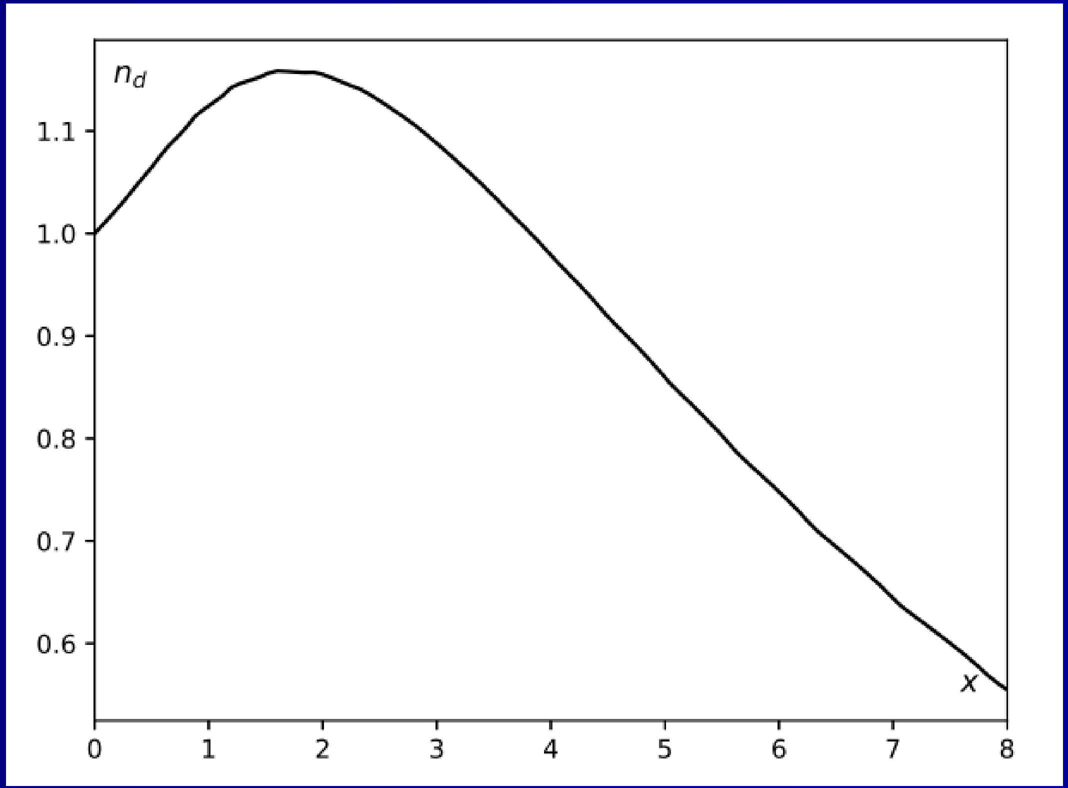


Each curve is a collection of parameters to satisfy:
x = a given value.

These eleven curves correspond to x = 0, 0.1, 0.2,……, 1.0

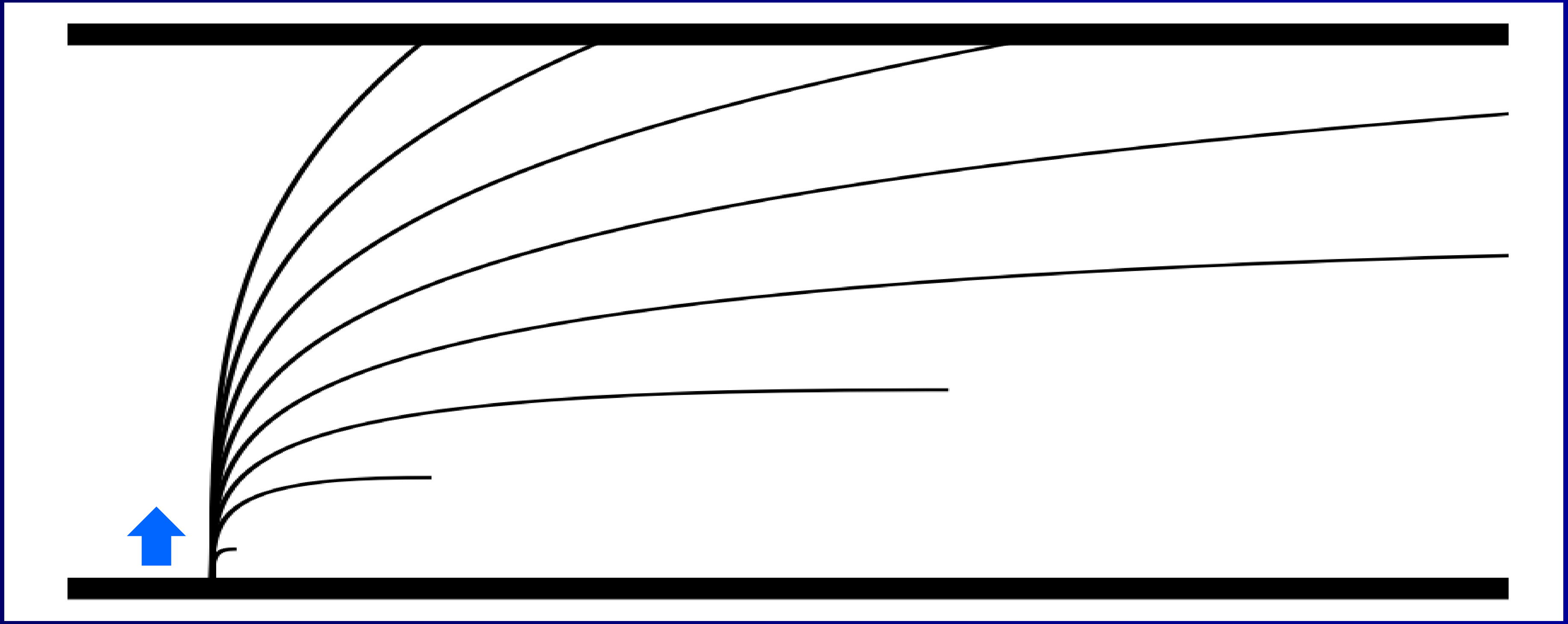
Horizontal injections: results

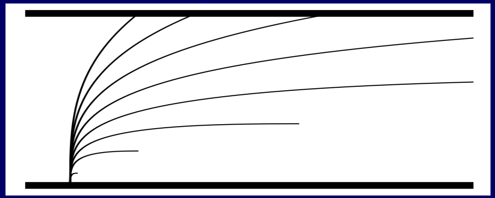




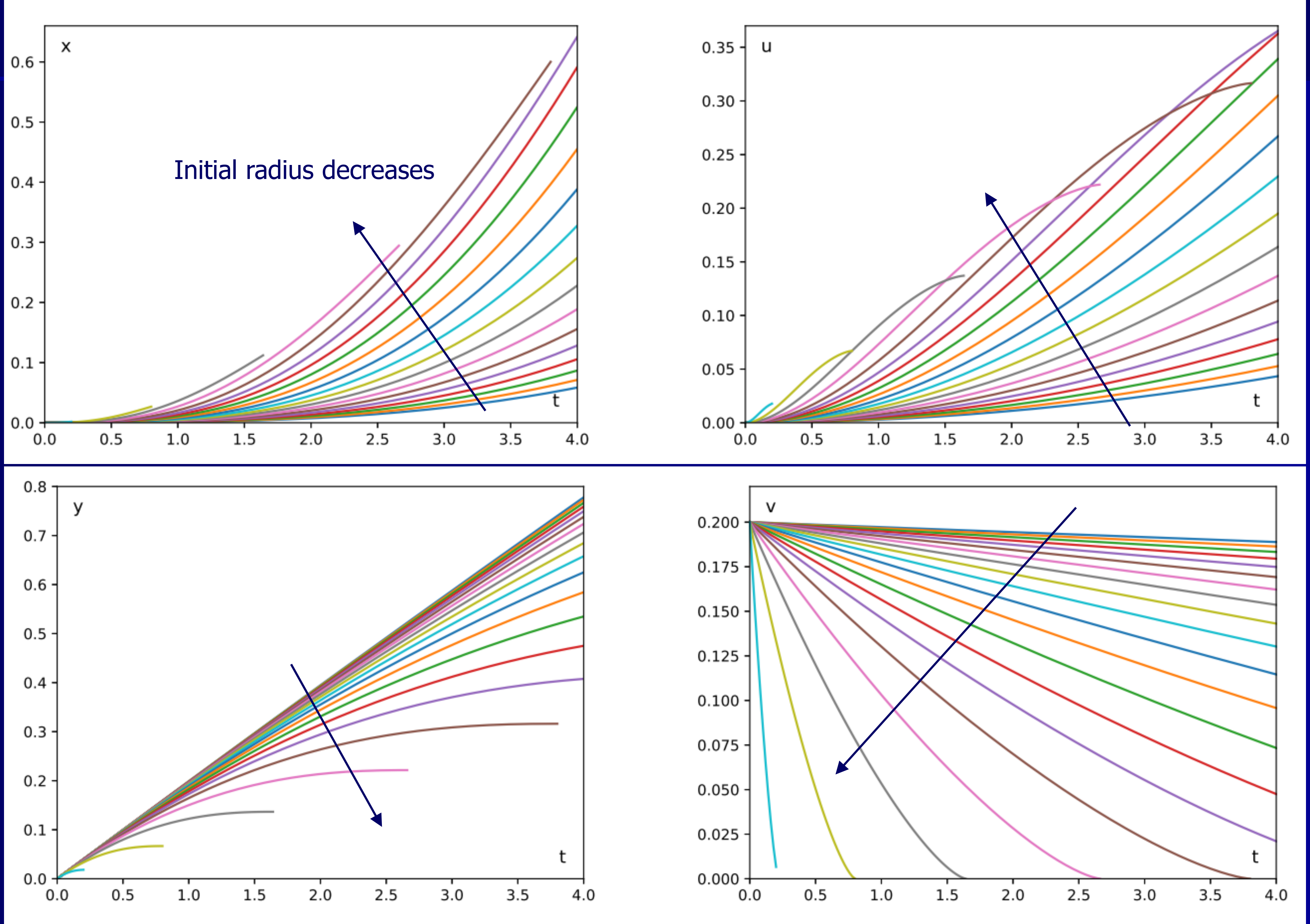
$$n_d = \int \tilde{n}_d dr_d$$

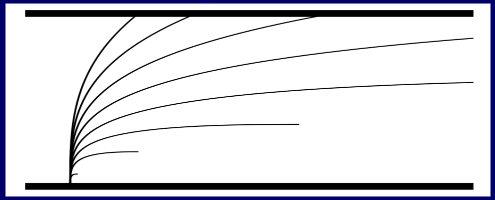
Example II:
Vertical injection into a hot planar Couette flow



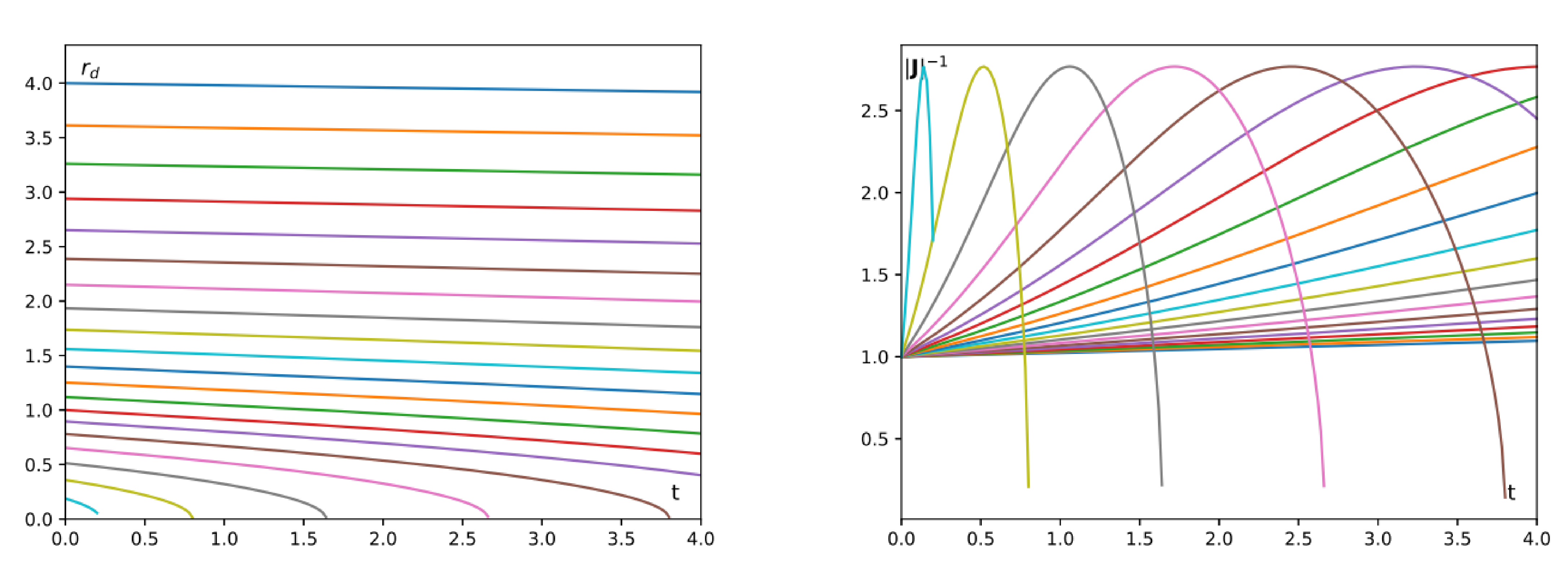


Vertical injection: results





Vertical injection: results



$$\tilde{n}_d = \tilde{n}_{d0} \|\tilde{\mathbf{J}}\|^{-1}$$

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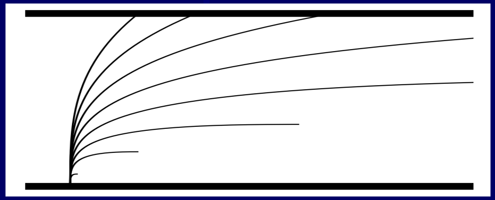
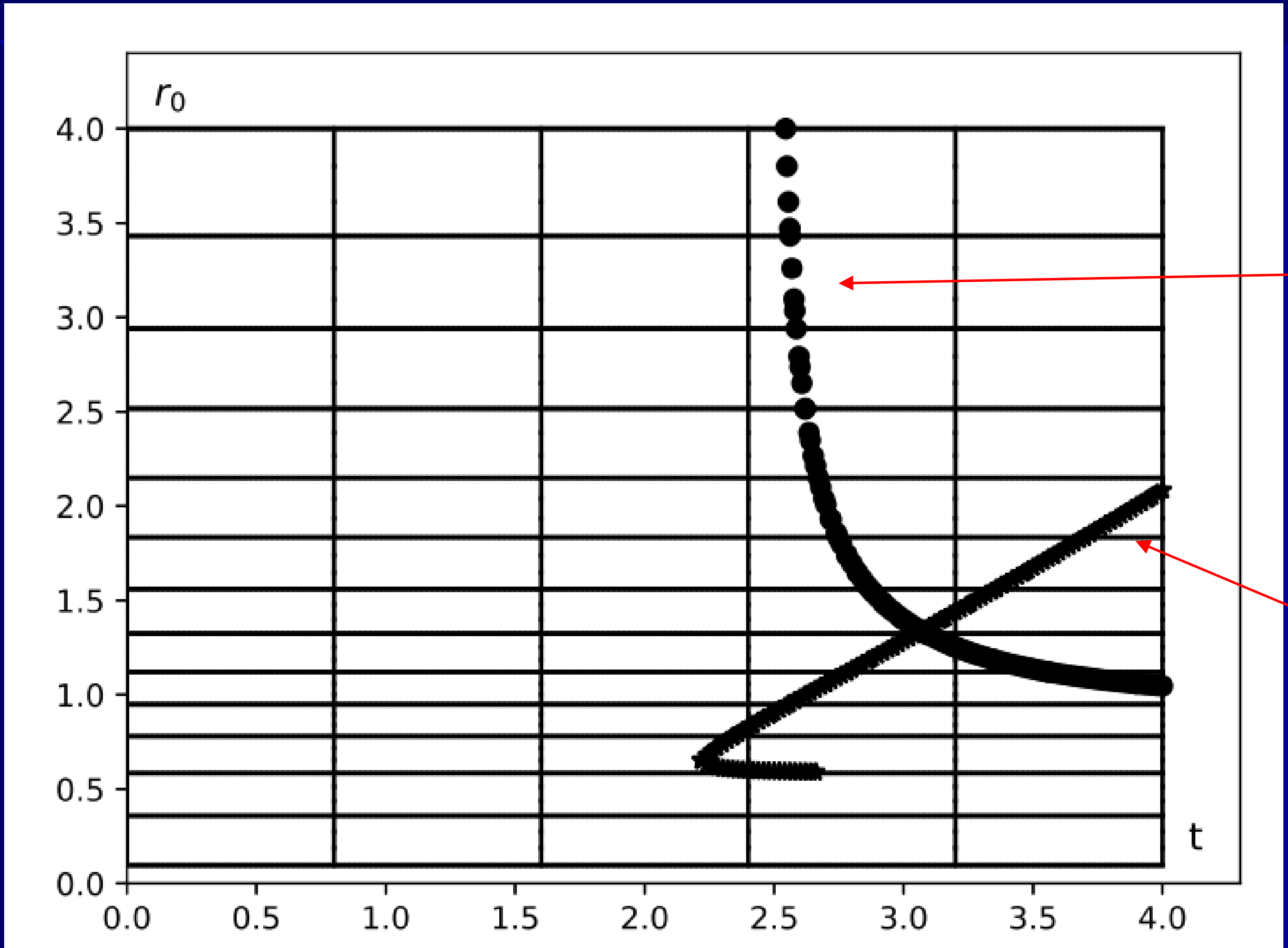
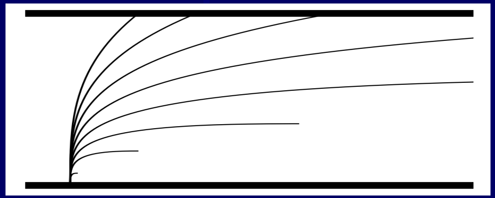


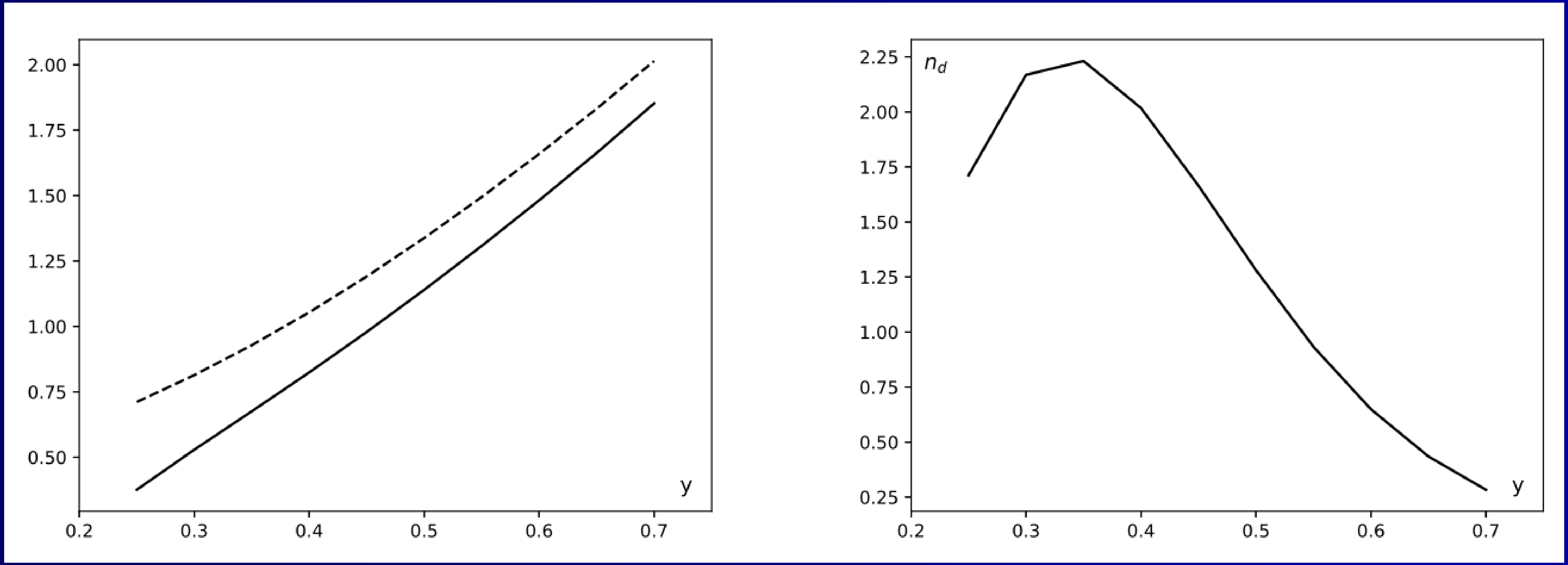


y = 0.5

x = 0.2

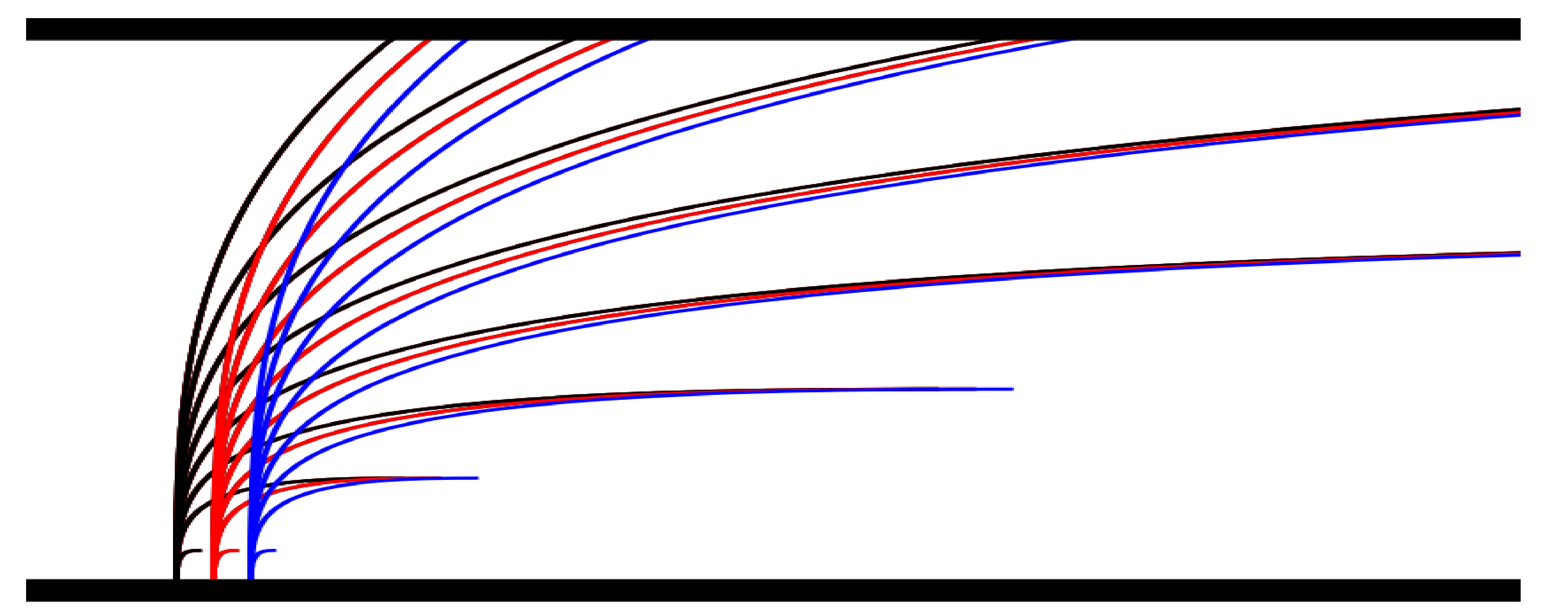


Vertical injection into a hot planar Couette flow

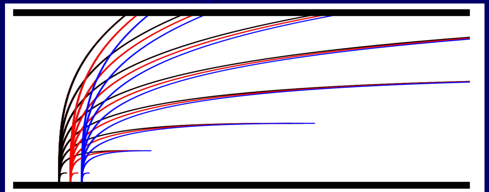


The distribution of r_d and n_d along the vertical line at x = 0.2

Example III:
Vertical injection over a finite-size injector



$$(\mathbf{x}, \mathbf{u}, \tilde{r}_d, \tilde{n}_d) = (\mathbf{x}, \mathbf{u}, \tilde{r}_d, \tilde{n}_d)(t, \tilde{r}_{d0}, x_0)$$



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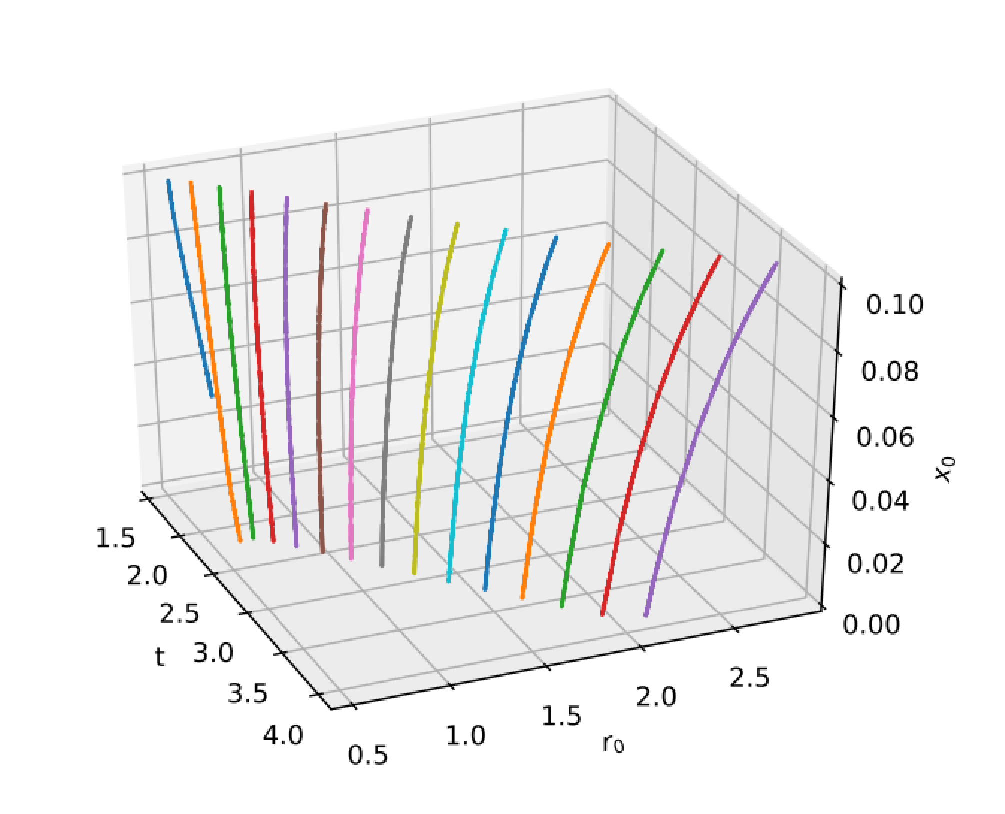
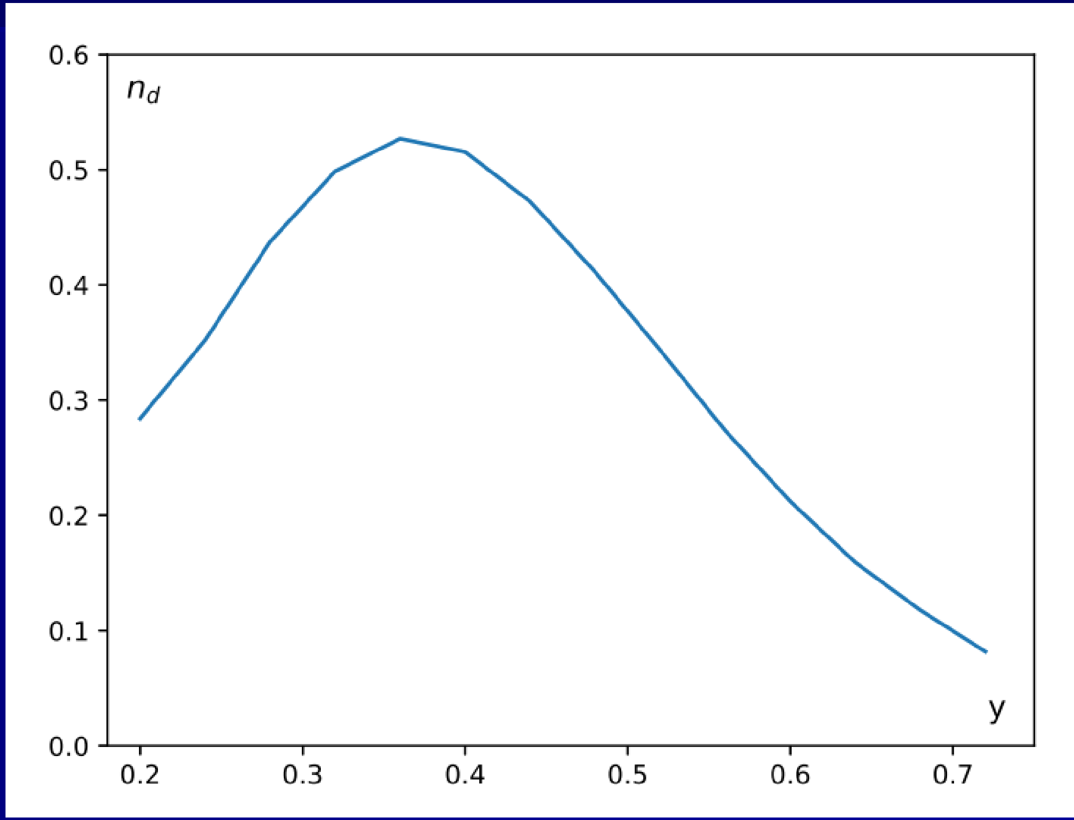
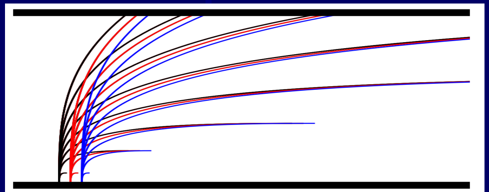




Conclusion:

- A generic interpolation scheme has been developed as an extension of Fully-Lagrangian Approach to calculate the droplet concentration of an evaporating polydisperse sprays;
- Preliminary computations show that this scheme has a good performance.

Thank you!



Vertical injection over a finite-size injector