

Flow instability, modelling & control

Second SIG Meeting in UK Fluids Network

Flow instability, modelling and control

28, August, 2018

Imperial College London

Programme

Schedule

Tuesday 28th August

10:00 Opening & Registration

Chair: Ati Sharma

10:30 Invited Lecture: Simon Ilingworth (Melbourne)
MODEL-BASED ESTIMATION OF FLUID FLOWS

11:20 Karim Shawk & George Papadakis (Imperial)
PRE-CONDITIONED ALGORITHM FOR THE EVALUATION OF SENSITIVITY OF TIME AVERAGES USING THE LEAST SQUARES SHADOWING METHOD

11:40 Elena Marensi, Ashley P. Willis (Sheffield) & Rich R. Kerswell (Cambridge)
STABILISATION AND DRAG REDUCTION OF PIPE FLOWS BY A FLATTENED BASE PROFILE

12:00 Geoffroy Claisse & Ati Sharma (Southampton)
FEEDBACK STABILIZATION OF A PLANE COUETTE FLOW INVARIANT SOLUTION

12:20 **Lunch**

Chair: Xuesong Wu

2:00 Sergei Chernyshenko (Imperial)
TOWARDS ACCELERATING TIME AVERAGING OF TURBULENT FLOWS

2:20 Giovanni Fantuzzi (Imperial)
SUM-OF-SQUARES POLYNOMIAL OPTIMISATION FOR HYDRODYNAMIC ANALYSIS

2:40 Ahmed A. Sheikh Al-Shabab & Mark A. Savill (Cranfield)
DIRECT NUMERICAL SIMULATION OF DETERMINISTIC TURBULENCE

3:00 Ubaid. Ali Qadri (Cambridge), Calum Skene & Peter J. Schmid (Imperial)
OPEN-LOOP CONTROL OF A SWIRLING JET: A WEAKLY NONLINEAR ANALYSIS

3:20 **Coffee Break**

Chair: Richard Hewitt

4:00 Xuesong Wu (Imperial)
NONLINEAR EVOLUTION AND ACOUSTIC RADIATION OF INSTABILITY WAVES/COHERENT STRUCTURES IN TRANSITIONAL/TURBULENT FREE SHEAR FLOWS

4:20 Robin Basso, Spencer Sherwin & Yongyun Hwang (Imperial)
THE INSTABILITY MODES OF VORTEX-INDUCED VIBRATIONS OF A BLUFF BODY AT LOW REYNOLDS NUMBER USING THE SPECTRAL ELEMENT METHOD

4:40 Lloyd Fung & Yongyun Hwang (Imperial)
SPATIO-TEMPORAL INSTABILITIES OF TILTED SHEAR FLOWS IN A STRONGLY STRATIFIED AND VISCOUS MEDIUM

5:00 **Closing**

Abstracts

Invited talk

Model-based estimation of fluid flows

Simon Illingworth

Department of Mechanical Engineering, University of Melbourne

One challenge for fluid mechanics is that any measurements are limited in space and in time. In this talk I consider model-based methods for estimating fluid flows from limited measurements. Specifically, model-based estimation is applied to the complex Ginzburg-Landau equation; to a turbulent channel flow in simulations; and to the flow past a cylinder in experiments. For each case the challenges of estimation are also discussed. While the focus throughout is on estimation, some aspects of control are also considered given the close relationship between the two.

A pre-conditioned algorithm for the evaluation of sensitivity of time averages using the Least Squares Shadowing method

Karim Shawki and George Papadakis
Department of Aeronautics, Imperial College London

Traditional sensitivity analysis methods often fail to compute the derivatives of long-time averaged quantities for chaotic dynamical systems (such as flow systems at relatively high Reynolds numbers). These derivatives are required for optimisation and uncertainty quantification applications, among many other applications.

The Least Squares Shadowing method (LSS) [1] has emerged as a robust solution to this problem. LSS requires the solution of a Karush–Kuhn–Tucker (KKT) system, which becomes prohibitively expensive to store and solve as the number of degrees of freedom of the system increases. To reduce costs, a recent approach [2] requires the computation of all positive Lyapunov exponents. Another approach based on a multiple shooting method [3] results in slow convergence. In this study we propose using a new iterative methodology to improve the performance. The new approach does not require the evaluation of Lyapunov exponents. Numerical experiments were conducted for 0D and 1D chaotic systems. The algorithm produced identical sensitivities as the original method, and converged significantly faster. The pre-conditioned system has a favourable clustering of eigenvalues, which is essential for scalability and efficient performance. The proposed method can be easily implemented using existing time-steppers for the constraint equations (in the case of flow systems, the Navier-Stokes) and the adjoint equations.

References

1. Wang Q, Hu R, Blonigan P. Least Squares Shadowing sensitivity analysis of chaotic limit cycle oscillations. *Journal of Computational Physics*, 267:210-224, 2014.
2. Angxiu N, Wang Q. Sensitivity analysis on chaotic dynamical systems by Non-Intrusive Least Squares Shadowing (NILSS). *Journal of Computational Physics*, 347:56-77, 2017.
3. Blonigan P, Wang Q. Multiple shooting shadowing for sensitivity analysis of chaotic dynamical systems. *Journal of Computational Physics*, 354:447-475, 2018.

Stabilisation and drag reduction of pipe flows by a flattened base profile

Elena Marensi¹, Ashley P. Willis¹ and Rich R Kerswell²

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²Department of Applied Mathematics and Theoretical Physics, University of
Cambridge

Recent experimental observations (Kühnen et al. 2018) show that a flattened streamwise velocity profile destabilises turbulence. By constructing variational problems that seek the minimal seed for turbulence, we show that flattening of the mean streamwise velocity profile also enhances stability of the laminar base flow, thus potentially preventing the transition to turbulence. To mimic the presence of a baffle that locally flattens the base profile, we apply a localised force. In order to generate initial conditions that may be considered to model ambient disturbances, we first compare the (unforced) transition behaviour of the minimal seed with several forms of randomised initial disturbances in the range $Re=2400$ to 10000 . Scalings $E_c \sim Re^{-\gamma}$ are obtained with γ in the range $2-3$ for different forms of disturbance, with $\gamma \approx 2.8$ for the minimal seed. We find that the energy of the minimal seed after the Orr and oblique phases is close to that of the localised random disturbance, so that in this sense the minimal seed after the oblique phase is a reasonable proxy for measuring transition thresholds. Feeding simulations in the presence of localised forcing with the initial conditions generated above, we continue to find increases in E_c , i.e. enhanced stability of the laminar flow, and destabilisation of turbulence. Drag reduction is found even when the turbulence is not fully relaminarised by the forcing.

Feedback stabilization of a Plane Couette Flow invariant solution.

Geoffroy Claisse and Ati Sharma
Engineering and the Environment, University of Southampton

Linear state-feedback control is able to relaminarise a plane channel flow with full-flow information and actuation (e.g. Bewley 1998; Sharma 2011). This talk will discuss extending full-information state-feedback control, limited to wall actuation, to stabilize an exact solution of the Navier-Stokes equations (the Nagata lower-branch). To achieve this we must linearise the Navier-Stokes equations around a steady three-dimensional non-laminar solution to obtain a new reduced model, called Orr-Sommerfeld Squire Extended model (OSSE), in order to solve the high-dimensional Algebraic Riccati Equation (ARE) used in the controller synthesis step. The OSSE model was able to reproduce the eigenvalues published by Gibson 2008 for different steady solutions. A Linear Quadratic Regulator (LQR) actuated by wall-transpiration was designed with the OSSE model. A modal controllability study indicated that the three unstable modes of EQ1 were all controllable with wall transpiration. Direct solution of the ARE, and thereby the optimal control law, is possible, but expensive. Remedies such as the Adjoint of the Direct-Adjoint method (Pralits and Luchini 2010, Semeraro 2013) and Krylov subspace methods (Jaimoukha 1995, 1997) are under investigation. The implementation of the control law in a CFD simulation (Heins 2015) is underway, and other controller types might follow.

Towards accelerating time averaging of turbulent flows

Sergei Chernyshenko

Department of Aeronautics, Imperial College London

In many practical applications the main goal of numerical modeling of a dynamical system is to obtain the long-time average of a certain set of parameters, such as, for example, the lift and drag force acting on an aircraft. The solutions of large nonlinear systems of equations often exhibit a chaotic behaviour. When the amplitude of the fluctuation of the quantity of interest is large, or when a very high accuracy is required because the effect to be investigated is small, obtaining time-averaged quantities with sufficient accuracy requires large averaging intervals, and thus, expensive numerical calculations.

The method we propose amounts to replacing the quantity being averaged with another quantity having the same average but fluctuating less. This is achieved using some of the ideas of the recently proposed method of bounding time averages (Chernyshenko et al., 2014, *Phil. Trans. Roy. Soc. A*, Vol: 372). Namely, one of the key elements of that method is that for any differentiable function $V(x)$, where x is the vector describing the state of the dynamical system, the infinite time average of $dV(x(t))/dt$ is zero provided that the trajectory of the dynamical system is bounded, which is always the case when infinite time averaging is meaningful. Hence, rather than numerically averaging the quantity of interest, which we will denote F , one can average $F+dV/dt$, for any V . The function $V(x)$ can be represented as a linear combination of a finite set of a suitably chosen basis functions. The coefficients of the linear combination can then be easily optimized to minimize the fluctuation of the finite-time average approximation of the infinite-time average, using two-point correlations of the basis functions and the quantity of interest, which correlations can be calculated during the direct numerical simulation of the dynamical system. Then the time-average of $F+dV/dt$ will converge faster than the time average of F . So far, this approach was tested on the Lorenz attractor and gave promising results.

Sum-of-squares polynomial optimisation for hydrodynamic analysis

Giovanni Fantuzzi

Department of Aeronautics, Imperial College London

Direct nonlinear analysis of fluid flows is notoriously challenging, due to the lack of exact analytical methods and the computational cost of fully resolved numerical simulations. In some cases, however, one can use an indirect approach, which does not require solving the governing equations. A well known example is that of Lyapunov stability analysis: the nonlinear stability of a laminar flow to arbitrary perturbations can be established by constructing a Lyapunov function, that is, a positive definite function of the perturbations that decays in time. Similarly, it has recently been shown that asymptotic or time-averaged quantities (e.g., kinetic energy or energy dissipation), as well as certain measures of nonlinear disturbance amplification, can be estimated through the construction of suitable "auxiliary" functions of the flow variables. A crucial observation is that, while Lyapunov or auxiliary functions are difficult to construct analytically, they can be searched for through the solution of optimisation problems involving sum-of-squares polynomials. These, in turn, can be solved numerically with efficient algorithms for semidefinite programming. Rigorous quantitative estimates for the properties of fluid flows can therefore be obtained without resorting to direct numerical simulations, promising a reduction in computational complexity compared to current practice. This talk will give an overview of recent advances in the application of sum-of-squares polynomial optimisation to hydrodynamic analysis, and results for some specific hydrodynamic systems will be showcased.

Direct Numerical Simulation of Deterministic Turbulence

Ahmed A. Sheikh Al-Shabab and Mark A. Savill

School of Aerospace, Transport and Manufacturing, Cranfield University

Direct numerical simulation is conducted to investigate recent experimental findings (Borodulin et. al. 2011) regarding the repeatability of coherent structures in the late transitional and early turbulent stages of a flat plate boundary layer under a controlled forcing. This so called 'Deterministic Turbulence', if proven to be an attribute of turbulence in nature, allows the time and location of appearance of turbulent structures to be predetermined, thus enabling a novel approach to active boundary layer control and advancing our theoretical understanding of turbulence. The DNS code is initially validated against standard benchmark test cases, such as Poiseuille flow and Taylor Green vortex, then the boundary layer case is gradually built starting with a two dimensional Blasius laminar boundary layer. A pulsating jet point source is then added to monitor the development of instabilities in the two and three dimensional cases, shown in Figures 1 and 2 respectively. Having chosen a forcing frequency in the stable range, the expected decay of streamwise and normal velocity disturbances is observed, while the spanwise velocity disturbance initially grows before it also starts decaying. These results are in agreement with similar tests in the literature and help establish confidence in the code to proceed with the amplified disturbance case. Further simulations are being carried out with white noise added to the forcing signal (as in the experiments) to test the effect of different levels of uncertainty on the coherence between different flow regions and the development of coherent transitional and turbulent structures in the boundary layer.

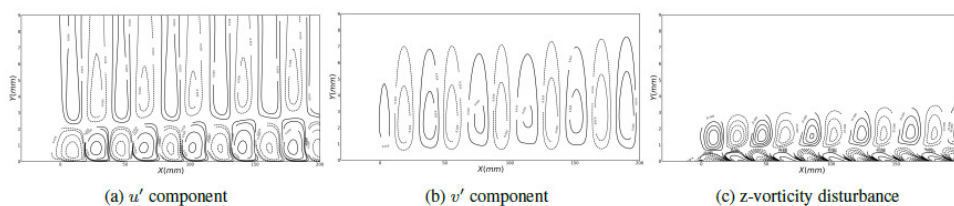


Figure 1: Forced two-dimensional boundary layer

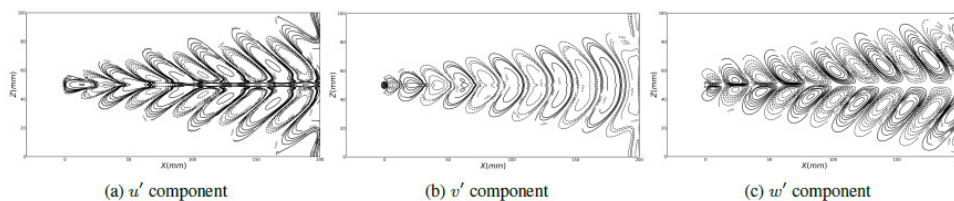


Figure 2: Forced three-dimensional boundary layer

Open-loop control of a swirling jet: a weakly nonlinear analysis

Ubaid Ali Qadri¹, Calum Skene² & Peter J. Schmid²

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Swirling jets are commonly used in a variety of industrial applications, such as gas turbine combustion, to improve mixing. At sufficiently high values of swirl, the jet experiences vortex breakdown, and, in many cases, a large-scale unsteady helical mode is known to develop downstream of the jet inlet. This mode is a manifestation of a global instability.

We explore the manipulation of this instability by an axisymmetric forcing, using a weakly nonlinear framework to allow for the interaction of the axisymmetric forcing with the nonaxisymmetric instability. This weakly non-linear manipulation is first demonstrated on an incompressible flow and then generalised for a compressible reacting case. The effect of the axisymmetric forcing is investigated for its ability to suppress the amplitude of the precessing instability, and the analysis is then extended to consider the effect of non-axisymmetric forcing.

Nonlinear evolution and acoustic radiation of instability waves/coherent structures in transitional/turbulent free shear flows

Xuesong Wu

Department of Mathematics, Imperial College London

Free shear flows, such as mixing layers, jets and wakes, are inviscidly unstable due to their inflectional velocity profiles. Instability modes, which are usually excited by external perturbations, amplify on the shear flow, leading to vortex roll-up and randomization in the nonlinear stage. Interestingly, in turbulent state free shear flows exhibit a high degree of order, characterised by the prevalent presence of the so-called coherent structures, the most striking of which are 'Brown-Roshko rollers'. These structures bear remarkable resemblance to instability modes, raising the prospect that their dynamics might be understood and predicted by instability theory.

Instability waves and coherent structures are known to be dynamically significant for entrainment and mixing, noise generation as well as for turbulence modelling. In this talk, I will present a unified nonlinear critical-layer theory to describe the development of instability modes/coherent structures on transitional/turbulent free shear layers. The effects of non-parallelism and small-scale fluctuations are accounted for. The theory predicts nonlinear saturation, randomisation through a generalized side-band instability mechanism and formation of 'Brown-Roshko rollers'. Furthermore, by analyzing the far-field asymptotic behaviour of the coherent structures, we are able to explain and predict, on the first-principle basis, how coherent structures radiate sound.

In search of the instability modes of vibrations of a bluff body at low Reynolds number, employing a spectral/hp element framework and coordinate transformation method.

Robin Basso, Spencer J. Sherwin and Yongyun Hwang
Department of Aeronautics, Imperial College London

The prediction of the motion of a bluff body in a continuous flow is still not well understood, although several analytical models are available in the literature in the case of an attached flow. We investigate here a better understanding of the phenomena of separated flow around a bluff body by the use of numerical approaches.

This study presents a two-dimensional investigation of the instability modes of vibrations (in both torsional and transversal directions) of a free-to-oscillate cylinder of diameter D fitted to a free-to-rotate splitter plate of length L , immersed in a constant flow of magnitude U .

A Spectral hp finite elements method is employed to carry out the numerical simulations, as well as a coordinate transformation method in order to consider the fluid structure interaction problem. The geometrical characteristic of the body and the flow regime are determined such as the length of the flat plate is chosen as the smallest for which we do not observe any turn angle during the steady state, and the Reynolds number is determined as the one for which there is no structural vibrations yet. For these values, a linear stability analysis of the base flow is computed and the modes are extracted.

Spatio-temporal instabilities of tilted shear flows in a strongly stratified and viscous medium.

Lloyd Fung and Yongyun Hwang

Department of Aeronautics, Imperial College London

It is well known that stratification can stabilise shear flow. In a vertical shear flow, the Miles-Howard's criterion firmly indicates that flow should be stable if the local Richardson number is greater than one fourth. However, a recent inviscid stability analysis showed that if shear is tilted with a non-zero angle from vertical, an instability can arise even under very strong stratification.

In the present study, we perform a linear stability analysis of tilted parallel shear flow in a viscous fluid. In the limit of very strong stable stratification, it is theoretically derived that the linear stability equation is given in the form of the Orr-Sommerfeld equation on the barotropic plane. It follows that the most unstable mode in this regime should always become barotropic. The barotropic layers are vertically correlated by viscous shear, which is consistent with past research. The most unstable mode remains to be two-dimensional but invariant in the spanwise direction, in spite of the misalignment between the invariant direction and the stratification direction.

A spatio-temporal stability analysis is subsequently performed for a tilted parallel wake flow at low Reynolds numbers to confirm the theoretical result consistent with a recent experimental observation, where the primary instability near the critical Reynolds number was found to be a two-dimensional inflectional type (i.e. Karman vortex shedding). The numerical analysis with the full set of equations reveals that exactly the same qualitative behaviour is reproduced with the experimental observation.